

Hiding Lorentz Invariance Violation with MOND

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Hořava-Lifshitz gravity is an attempt to construct a renormalizable theory of gravity by breaking the Lorentz Invariance of the gravitational action at high energies. The underlying principle is that Lorentz Invariance is an approximate symmetry and its violation by gravitational phenomena is somehow hidden to present limits of observational precision. Here I point out that a simple modification of the low energy limit of Hořava-Lifshitz gravity in its non-projectable form can effectively camouflage the presence of a preferred frame in regions where the Newtonian gravitational field gradient is higher than cH_0 ; this modification results in the phenomenology of MOND at lower accelerations. As a relativistic theory of MOND this modified Hořava-Lifshitz theory presents several advantages over its predecessors.

I. INTRODUCTION

A recent theoretical motivation for a universal preferred frame is provided by a modern attempt to construct a renormalizable quantum theory of gravity. It has been known for some time that the presence of higher derivatives of the metric tensor in the field equations can make the theory renormalizable [1]. However, in a covariant theory, higher spatial derivatives also mean higher time derivatives and such theories tend to be unstable. To solve this problem, Hořava [2] has proposed that Lorentz Invariance (LI) is broken at high energies by an additional geometrical structure, a preferred space-like foliation that splits space from time. Then it is possible to construct a renormalizable theory with higher spatial derivatives while maintaining only two time derivatives. Although the higher derivative terms become negligible at low energies, the preferred frame is fundamental and present at all energies; it is *assumed* that the LI violating terms in the action become negligible at low energies in order to satisfy phenomenological constraints. The LI violation leads to an extra scalar degree of freedom which in the original “projectable” Hořava-Lifshitz Gravity (HLG) presents problems in principle. Blas, Pujolas and Sibiryakov [3, 4] (BPS) have shown that these problems are eased by taking a particular form of Hořava-Lifshitz gravity, the so-called non-projectable form in which an additional term, second order in the preferred frame, is added to the action to provide stability. But the central point remains: renormalization in the context of HLG requires that we live in a Universe in which LI is only an approximate or apparent symmetry; the Universe contains a fundamental preferred frame that is somehow hidden to present experimental accuracy. My purpose here is to consider how this camouflage may be accomplished in terms of a simple modification of low energy HLG – a modification leading to MOND phenomenology.

I outline the structure of an acceleration-based modification of Hořava-Lifshitz gravity which may be viewed as a special case of generalized modified vector-tensor or Einstein-Aether (EA) theories [6–8]. I describe its advantages as a relativistic theory of MOND and stress that such a theory can be consistent with local and cosmo-

logical constraints on GR including preferred frame effects, the absence of Cerenkov losses on high-energy cosmic rays, and nucleosynthetic limits on the cosmological value of G . The theory provides enhanced gravitational lensing about distant astronomical objects (as though by dark matter) without the construction of a second disformally related metric. Therefore, independent of the connection with HLG, this modified EA theory, as a relativistic theory of MOND, has relatively few additional ad hoc elements.

II. BACKGROUND

In Hořava-Lifshitz gravity LI is broken at high energies by the presence of a preferred foliation of 3-D surfaces in space-time. In the BPS version, the splitting is dynamical; i.e., level surfaces of a dynamical scalar function C define the foliation (BPS call this the “khronon” field). The foliation persists to low energies but presumably with LI breaking terms strongly suppressed.

It is useful to define a unit vector which is the normalized khronon field gradient,

$$A_\mu = \frac{C_{,\mu}}{\sqrt{-g^{\alpha\beta} C_{,\alpha} C_{,\beta}}} \quad (1)$$

Then at low energies the modified BPS non-projectable form of HLG may be written in covariant form as

$$S_{HLG} = \frac{c^4}{16\pi G} \int [R - L_C] \sqrt{-g} d^4x \quad (2)$$

with

$$L_C = (\lambda - 1)(\nabla_\mu A^\mu)^2 + \alpha a_\mu a^\mu \quad (3)$$

where $a^\mu = A^\rho \nabla_\rho A^\mu$ is the acceleration of curves normal to the foliation surface. The invariant $a_\mu a^\mu$ is the term added by BPS for stability of the scalar degree of freedom with α as the dimensionless constant characterizing its contribution to the energy-momentum.

This is recognized as a subclass of EA theories [7] where

$$L_{EA} = M^{abmn} \nabla_a A_m \nabla_b A_n \quad (4)$$

with

$$M^{abmn} = c_1 g^{ab} g^{mn} + c_2 g^{am} g^{bn} + c_3 g^{an} g^{bm} + c_4 A^a A^b g^{mn} \quad (5)$$

As written here, the only non-zero couplings are $c_2 = \lambda - 1$ and $c_4 = \alpha$. Given the definition of A_μ as a scalar field gradient, the theory would appear to contain dangerous higher derivatives of the field C . However, this is avoided when one transforms to the special frame where C becomes the time coordinate [7]. Then one may easily show that

$$A_\mu = \delta_{\mu C} (-g^{CC})^{-\frac{1}{2}} \quad (6)$$

and

$$a_i = [\ln(N)]_{,i} \quad (7)$$

with $N = \sqrt{-g_{CC}}$, the lapse function in the language of ADM 3+1 formalism. Here “ i ” refers to spatial derivatives in the preferred frame.

The first term in eq. 3 involves only time derivatives of the metric, so considering the static case, we may take $\lambda \approx 1$ (reconsidered below). Then in the preferred frame the action of the BPS extended HLG may be written simply as

$$S_{HLG} = \frac{c^4}{16\pi G} \int \sqrt{-g} [R - \alpha \frac{N_{,i} N_{,i}}{N^2}] d^4 x \quad (8)$$

In this frame the Einstein equations become

$$R_{ij} - \frac{1}{2} g_{ij} R = \frac{8\pi G T_{ij}}{c^4} + \alpha \left[\frac{N_{,i} N_{,j}}{N^2} - \frac{1}{2} g_{ij} \frac{N_{,k} N_{,k}}{N^2} \right] \quad (9)$$

Keeping in mind that in this frame the square of the lapse is identified with g_{CC} (L. Blanchet, private communication), the equation for the lapse is given by

$$g^{CC} R_{CC} - \frac{1}{2} R = \frac{8\pi G}{c^4} g^{CC} T_{CC} + \alpha \frac{[g^{ij} N_{,i}]_{,j}}{N} - \frac{\alpha}{2} \frac{N_{,i} N_{,i}}{N^2} \quad (10)$$

One should note that this is the field equation written in the preferred frame and not in covariant form. Covariant equations may be found in references [7, 20]. It is also noteworthy that in this frame the new degree of freedom, the scalar Khronon field, does not appear explicitly (it is the time coordinate); the LI breaking due to HLG is apparent as revised Einstein equations.

I take small perturbations about Minkowski space of the form

$$g_{ij} = \delta_{ij} (1 - 2\psi) \quad g_{CC} = -(1 + 2\phi) \quad (11)$$

where the scalars ϕ and ψ are the usual Newtonian potentials. This implies that $N = 1 + \phi$. Then in the static case,

$$\psi = \phi \quad (12)$$

due to the absence of first order contributions to the source of G_{ij} , and

$$2\nabla^2 \psi - \nabla \cdot [\alpha \nabla \phi] = 8\pi G \rho / c^2 \quad (13)$$

where α is included inside the gradient operator because in the modified version, considered below, it is dependent upon spatial position in the preferred frame. Note that this is equivalent to the two-field non-relativistic theory described by the Lagrangian

$$L_{NR} = 4\nabla\phi \cdot \nabla\psi - 2\nabla\psi \cdot \nabla\psi - \alpha \nabla\phi \cdot \nabla\phi + \frac{16\pi G}{c^2} \phi \rho \quad (14)$$

As stressed by several authors [9–11] the essential difficulty concerns the LI violating parameters, α and $\lambda - 1$. There is nothing in the theory that requires these to become small enough to be consistent with the very tight phenomenological constraints on deviations from GR (absence of gravitational preferred frame effects in the Solar System, extragalactic propagation of high energy cosmic rays, variations between local and cosmological values of G , binary pulsar constraints). Below I describe a modification of HLG which can fit this bill; a modification attached to gravitational acceleration rather than energy.

III. MODIFIED HOŘAVA-LIFSHITZ GRAVITY

Modifications of EA theories as possible relativistic extensions of MOND have been considered by Zlosnik, Ferreira and Starkman [8] (ZFS) who substitute for this general vector field Lagrangian (eq. 5) a specific function of that Lagrangian, $F(L_{EA})$. It would seem a trivial step to modify HLG in the same way by taking an appropriate function of L_C in eq. 3. But this would be equivalent to setting $\lambda - 1 \propto \alpha$; here I write a more general modification by adding two potentials to the HLG Lagrangian, i.e.,

$$L_C' = L_C + V(\alpha) + U(\lambda') \quad (15)$$

where $\lambda' = \lambda - 1$. This is equivalent to replacing the two vector field invariants in eq. 3 by separate functions of these invariants. The parameter α is then given by the solution to

$$\frac{dV}{d\alpha} = -\phi_{,i} \phi^{,i} \quad (16)$$

and λ' by

$$\frac{dU}{d\lambda'} = -\sqrt{\frac{g^{CC}}{g}} \frac{dg}{dC} \quad (17)$$

where g is the determinant of the metric tensor (recall that C is the time coordinate).

I introduce the modification as added potentials (as in TeVeS) rather than taking non standard kinetic terms because this leaves unchanged the form of the LI breaking invariants in the HLG Lagrangian (eq. 3); the kinetic terms are standard. In any case, there are no underlying principles here; such ad hoc modifications of scalar or vector field invariants are not new and have generally

been a basis for relativistic MOND (TeVeS, for example) or dynamical dark energy (k-essence, for example).

Given the equality of the Newtonian potentials (eq. 12), in the weak field static case the equation for ϕ (eq. 14) becomes

$$\nabla \cdot [\mu(|\nabla\phi/a_0|)\nabla\phi] = 4\pi G\rho \quad (18)$$

with $\mu = 1 - \alpha/2$. This is recognized as the Bekenstein-Milgrom [12] non-relativistic MOND field equation.

For the desired phenomenology the potential must be normalized by the single new physical constant – the MOND acceleration parameter a_0^2 . In the Newtonian limit, where $\alpha \rightarrow 0$, an appropriate form for the potential would be

$$V(\alpha) = \frac{(2^p)}{p-1} \alpha^{(1-p)} [a_0^2] \quad (19)$$

where $p > 0$ (the special case of $p = 1$ corresponds to a logarithmic potential). In the MOND limit where $\alpha \rightarrow 2$

$$V(\alpha) = \frac{2}{3} (1 - \alpha/2)^3 [a_0^2] \quad (20)$$

Note that the natural scale of the potential is a_0^2 which would be the approximate value of any cosmological term (H_0^2).

The remarkable aspect of Hořava-Lifshitz Gravity is the equality of the two Newtonian potentials (eq. 12). This implies that the relation between the weak-field force and the deflection of photons is identical to that of GR without the construction of two disformally related metrics, gravitational and physical, as in TeVeS [13]. This fact was first appreciated by ZSF in connection the modified EA theories.

Because the potential $V(\alpha)$ may be chosen such that the theory is arbitrarily close to GR in the Solar System, one might expect that the various post-Newtonian parameters (PPN) may be pushed to within current experimental accuracy of their GR values. But the advantage of considering modified HLG as a generalized EA theory is that constraints on the coefficients, c_i , have been worked out for EA theory [14]. In particular, the inevitable preferred frame effects are parameterized by

$$\alpha_1^{PPN} = -4\alpha \quad (21)$$

and, in the limit where α is small,

$$\alpha_2^{PPN} = -5\alpha/2 \quad (22)$$

For example, taking $V(\alpha) = -a_0^2 \ln(\alpha)$ where $\alpha \ll 1$ then, at the position of the earth in the Solar System, this gives $\alpha = 2.8 \times 10^{-16}$ or $\alpha_2^{PPN} = -7 \times 10^{-16}$ in the neighborhood of the earth, well below the observed constraint of $\alpha_2 < 10^{-7}$.

For time dependent problems, cosmology or wave propagation, it is necessary to consider the second parameter $\lambda' \neq 0$. A number of the results of modified EA theory [15] are directly applicable to the model suggested

here but with the restrictions relevant to non-projectable HLG. For an isotropic homogenous Universe (FRW), λ' is time-dependent and given by the solution to

$$9H^2 = -dU/d\lambda' \quad (23)$$

That is to say, λ' is a function of cosmic epoch (taking $\lambda' \propto \alpha$ in the spirit of the ZFS modification would imply that the cosmological value of α would be much smaller in the early universe – no MOND at earlier epochs). Normalizing $U(\lambda')$ by a_0^2 and defining

$$K = 9 \frac{H^2}{c^2 a_0^2} \quad (24)$$

one may write the LI violating term, in the language of modified EA theories, as $F(K) = \lambda' K$. Then the modified Friedmann equation given in [14] becomes

$$\left[1 + 3\lambda' \left(\frac{d \ln(\lambda')}{d \ln(K)} + \frac{1}{2}\right)\right] H^2 = \frac{8\pi G}{3} + \frac{U(\lambda')}{6} + \frac{V(\alpha)}{6} \quad (25)$$

where the cosmological potentials U and V are special to the present model.

If λ' is constant one recovers the well-known result [16]

$$G_c = G_N / (1 + 3\lambda'/2) \quad (26)$$

That is to say, the cosmological value of G is generally less than the local Newtonian value. Observations of the abundances of light isotopes in the context of primordial nucleosynthesis constrain $|G_c/G_N - 1| < 0.13$. This places restrictions on $U(\lambda')$. Taking a generic form $dU/d\lambda' = -\lambda'^{-p}$ we find that if $p = 2$ then $G_c = G_N$ with no modification to standard Friedmann expansion (apart from the cosmological potentials). If $p = 1$, corresponding to a logarithmic potential ($U(\lambda') \propto -\ln(\lambda')$) then $G_c = G_N$ but with a cosmological term on the order of $(a_0/c)^2$. Other values of p yield quintessence. Therefore, the theory possesses sufficient flexibility to avoid the nucleosynthetic constraint and to embody dark energy as well as MOND.

I stress that the dark energy is not a necessary aspect of the theory. For example, one may set $\gamma = 0$ and let α be a function of the invariant it multiplies ($N_{,i} N^{,i} / N^2$ in the preferred frame) – a function designed to yield MOND in the low acceleration limit 0. This would, in the absence of fluctuations, yield a completely standard cosmology with no cosmological constant or dark energy, only the gravitational attraction would be modified. However, while dark energy can be fine-tuned away, it would seem to be a positive attribute that dark energy of the right magnitude (a_0^2) is most naturally included.

Properly speaking, HLG is a restricted case of EA theory in which the vector field A^μ constrained to be orthogonal to surfaces of constant C (eq. 1); it is hypersurface orthogonal [7]. For wave propagation this means that, in addition to the usual tensor mode of GR, only one longitudinal (scalar) mode can propagate, and not the additional transverse modes (vector) of the general EA

theory. For the hypersurface orthogonal theory considered, the usual gravitational radiation, the tensor mode, propagates at the speed of light. The propagation velocity of the scalar mode is

$$c_s^2 = \left(\frac{\lambda'}{3\lambda' + 2} \right) \left(\frac{2 - \alpha}{\alpha} \right) \quad (27)$$

Independently of the value of λ' the scalar waves must propagate at a velocity below the speed of light in the cosmological limit where $\alpha \rightarrow 2$. This would previously have been considered fatal for the theory because of the perceived Cerenkov constraints. But recently Milgrom [17] has pointed out that the near field of a highly relativistic particle is in the Newtonian regime (acceleration greater than a_0) which implies a stopping distance due to gravitational Cerenkov comparable to the Hubble scale; there is no problem with Cerenkov losses for theories that approach GR in the regime of high accelerations. Therefore, the theory remains valid even given subluminal longitudinal wave propagation.

Note that radiation damping of compact binary pulsar systems agree with the predictions of GR to within one percent. This means that $\alpha, \lambda' < 0.01$ [18], a requirement easily met by the modified HG theory in the high acceleration regime.

IV. CONCLUSIONS

It has been demonstrated [8] that modified EA theories can form a relativistic basis for MOND. I emphasize here that this includes non-projectable HLG as a restricted EA theory. HLG in no sense implies MOND; the theory must be modified by making the coupling of the additional BPS invariant (α) dependent upon the invariant (either by adding a potential $V(\alpha)$ or taking a specific function of the invariant as in modified EA theories).

As relativistic generalizations of MOND, these modified EA or HLG theories are appealing. There are relatively few additional parameters apart from a_0 ; in fact the parameter α defines the MOND interpolating function and is required to have definite limits. Moreover, unlike TeVeS, such theories inevitably approach GR in the high acceleration limit. This means that the classical tests of GR can be readily satisfied at current levels of accuracy and, most importantly (following Milgrom), there is no Cerenkov constraint on the propagation of energetic cosmic rays. The theory is consistent with observations of cosmic gravitational lenses (i.e., additional deflection of photons by “phantom” dark matter) without the ad hoc construction of a disformally related physical metric.

Solar system and galaxy scale phenomenology can be consistent with this theory but constrain the form of the potential $V(\alpha)$. The cosmology is standard but with cosmological terms (dark energy), constant or evolving, of

approximately the correct magnitude (a_o^2). As in all EA theories, the cosmological G may differ from its local value, and the limits imposed by primordial nucleosynthesis constrain the form of the potential $U(\gamma')$. At present, the potentials are ad hoc; there are no a priori considerations which tell us what these potentials should be. Issues such as the viable cosmologies and the growth of fluctuations will be discussed in a later paper.

In a general sense, HLG requires that Lorentz invariance is not a fundamental symmetry but is violated by gravitational phenomenology. But why then is this LI violation not evident in the world around us as, for example, observable gravitational preferred frame effects? The MONDian modification provides one possible solution to this problem: the theory becomes GR to high precision in the high acceleration environment of the Solar System. The dynamical effects of the preferred frame are hidden by the modified HLG Lagrangian; MOND phenomenology occurs in the transition in the outer regions of galaxies between local dynamics described by GR and a preferred frame LI violating cosmology. This of course is speculative, and there are other possible theoretical bases for MOND such as Milgrom’s BiMOND [19]. However, the modified HLG is simple and efficient; it does not add many new elements, and it is connected to a well-motivated approach to quantum gravity.

The proposed theory is a viable relativistic extension of MOND, but there remain a number of issues to consider before this proposal could be taken as a viable extension of HLG, primarily connected with the MOND *Ansatz*: the variability of α and λ' through the addition of potentials $V(\alpha)$ and $U(\lambda')$. Since the possibility of power-counting renormalization is the motivation for breaking LI in Hořava-Lifshitz gravity, then one must ask if the assumption of acceleration-dependent couplings vitiates this attribute. My intuition is that it does not. Power-counting renormalization in this context requires that the operators in the action have dimension (number of spatial derivatives) of no more than six. This is certainly true of the α term in the MOND limit. This modification would seem only to affect the low energy properties of the theory. Questions of stability and/or ghosts should also be reconsidered, as well as the strong coupling limit, the energy above which the higher order terms in the full HLG action, become dominant. The possibility that α and/or λ' can be a dynamical fields might be considered; perhaps long wavelength oscillations in such fields could play a role as cosmic dark matter.

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